Optimization of Frame Topology Using Boundary Cycle and Genetic Algorithm*

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We report topological optimization of elastic frames by means of the boundary cycle used in algebraic topology and a genetic algorithm. In this study the optimum frame is defined as that in which the deformation at a point is minimal for a given weight limit. Members that have a tip which is not connected to other members, and increase the weight without making any mechanical contribution are neglected. The optimum topology is identified efficiently using a boundary cycle which yields a one-dimensional simplicial complex with no tips, satisfying the topological condition of no idle tip. The boundary cycle is derived from a chain and boundary operator, which plays the important role of decoding the genotype into the phenotype in the genetic algorithm, and is included in the string used in the genetic algorithm to represent the frame topology. The numerical examples are concerned with minimization of the deformation of two-dimensional frames subject to bending, and three-dimensional frames subject to torsion or expansion analyzed by the finite element method.

Key Words: Optimum Design, Computational Mechanics, Framed Structure, Topology, Boundary Cycle, Genetic Algorithm

1. Introduction

Structural optimization can be classified into the two categories of size optimization and topological optimization. Various methods of topological optimization have been developed recently[1],[2], but they are not as advanced as the methods of size optimization and a general method has yet to be developed.

In this paper we report topological optimization of two- and three-dimensional frames analyzed by the finite element method. A formulation is proposed based on algebraic topology for topological optimization of frame structures. A member with a tip that is not connected to another member does not contribute to the transmission of internal forces, and thus increases the weight of the structure without making any mechanical contribution. Absence of such tips is one necessary requirement of an optimum light-weight frame. It is feasible to identify a frame of minimum weight or minimum deformation in a set of frames which have no unconnected members. Such a set of frames can be generated by means of a boundary cycle[3]. The 1-boundary cycle used in algebraic topology is a one-dimensional simplicial complex without any tips, which satisfies the above-mentioned necessary condition. The usefulness of the boundary cycle for structural optimization was demonstrated in numerical examples concerned with finding the shortest boundary lines which partition a domain into several subdomains of equal area[4].

In this study a method of representing the topology of a frame by a combination of integers is proposed, and the optimum combination is identified using a genetic algorithm (GA). Numerical examples concerned with the minimization of deformation of a frame under the constraint of constant weight are given. Deformation of the frame is analyzed by the finite element method under the assumptions that the frame remains in the elastic state and the displacement and strain are small.

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2. Statement of problem

The topology of two- and three-dimensional frames is optimized. In this study the optimum frame is defined as that in which the sum of the displacements at the loading points is a minimum, with a weight equal to a certain prescribed value.

Figure 1 shows a ground structure for identifying the optimum two-dimensional frame which is fixed on a rigid wall and subject to a vertical force of 98.1 N at a point 1 m from the rigid wall. The aim is to minimize the vertical displacement at the loading point. Under the assumption that the optimum frame consists of some members of the ground structure shown by the solid lines in Fig. 1, the frame members are selected from those in the $1\times 1\mathrm{m}^2$ design domain shown in Fig. 1. The intersection points of the solid lines are called nodes.

In the case of three-dimensional frames, the members are arranged in a cube as shown in Fig. 2 so as to minimize the sum of the displacements of loading points A, B, C and D under the constraint condition of constant weight. The members of the optimum frame are selected from those shown in Fig. 3.

The cross section of all members is assumed to be a circle which has the same diameter in both the two- and three-dimensional frames since we aim at optimization of topology only. The case in which some tips of the frame member are loading points is not considered.

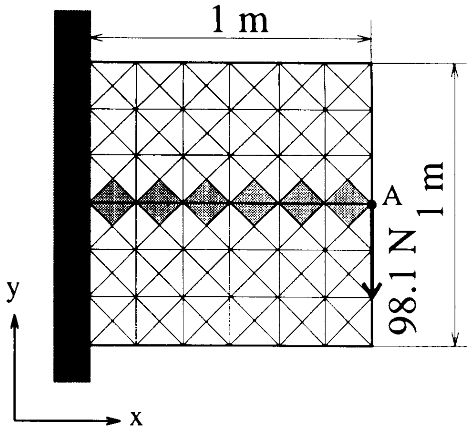

Fig. 1 Ground structure for optimization of two-dimensional frame

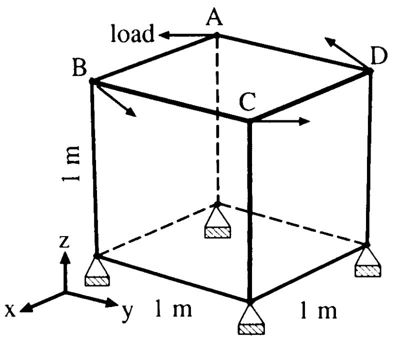

Fig. 2 Cube for optimization of three-dimensional frame

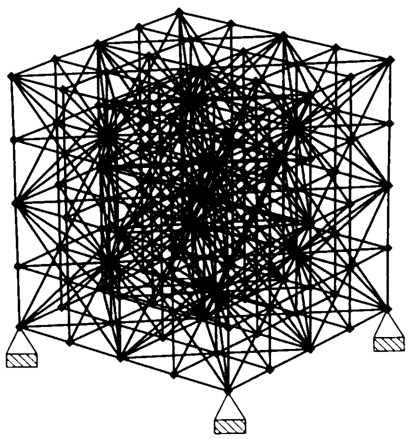
Fig. 3 Ground structure for optimization of three-dimensional frame

3. Method of coding topology using integers, and optimization by GA

3.1 Coding by boundary cycle

A boundary cycle is used to represent the topology of a frame in which there are no members that have a free tip, are not connected to other members, and do not contribute either to force transmission or to the frame stiffness. The characteristics of the boundary cycle are described in Ref. (4). The definition of the boundary cycle and related technical terms are summarized as follows. The topology of a frame can be represented succinctly by a string of integers using the boundary cycle, that is, a one-dimensional simplicial complex with no tips.

On the assumption that $a_i\,(i=0, \cdots, r)$ are independent points, the r-simplex that consists of the vertexes a_i is written as

$$x^r=(a_0, a_1, \cdots, a_r). \qquad (1)$$

A set of simplexes that satisfy the following conditions is called a simplicial complex which is denoted by K in the following.

1) When an arbitrary simplex belongs to K, its arbitrary face also belongs to K.
2) An intersection between two simplexes in K is a face common to each.
3) Each simplex in K is a face of the finite number of simplexes in K. This condition is called local finiteness.

The summation of integer coefficients $t^i(i=1, \cdots, m)$ multiplied by the r-simplexes is called an r-chain which is written in the form

$$c^r=t^1x_1^r+t^2x_2^r+\cdots+t^mx_m^r, \qquad (2)$$

where m denotes the number of r-simplexes in K. The operation of the boundary operator, ∂_r, on the r-simplex is defined as

$$\partial_r x^r = \sum_{i=0}^{r}(-1)^i(a_0, \cdots, \hat{a}_i, \cdots, a_r), \quad (3)$$

where $\hat{a}_i$ means that a_i is absent. The $(r-1)$-boundary cycle is the r-chain operated with ∂_r as given by

$$\partial_r c^r = t_1\partial_r x_1^r + t^2\partial_r x_2^r + \cdots + t^m\partial_r x_m^r. \quad (4)$$

The boundary operator has a feature $\partial_{r-1}\circ\partial_r = \{0\}$. This means that there is no edge of edge, and ensures that the 1-boundary cycle has no tip. Equation (4) indicates that once a certain r-dimensional simplicial complex is determined, the $(r-1)$-boundary cycle is specified by the integer coefficients t^i only. In other words, decision of the integer t^i results in the generation of a frame with no tip. Therefore, optimization of the frame topology is achieved by finding the optimum combination of integers t^i which are taken as a design variable.

A two dimensional complex that consists of the 2-simplexes shown in Fig. 1 is used for optimization of the two-dimensional frame. First, integer coefficients are allotted to each 2-simplex in order to make a 2-chain. Then the 1-boundary cycle is obtained by operating ∂_2 on the 2 chain. Nonzero integer coefficients are allocated to the gray 2-simplexes in Fig. 1 in order to ensure that there are members that connect the loading point and the rigid wall.

The simplicial complex used for optimization of the three-dimensional frame is generated by dividing the cube shown in Fig. 2 into tetrahedral 3 simplexes. The 2-boundary cycle is obtained in the same way as the 1-boundary cycle, that is, allocation of integer coefficients to the 3-simplexes and operation of ∂_3. In order to obtain the 1 boundary cycle, the absolute value of the integer coefficients of the 2-boundary cycle is replaced with unity, and ∂_2 is operated on this modified 2-boundary cycle. The replacement of the absolute value is necessary to prevent the 1-boundary cycle from becoming zero, because $\partial_2\circ\partial_3$ is $\{0\}$ unless this procedure is carried out.

3.2 Genetic algorithm

The total number of combinations of t^i is so large in most cases that it is necessary to choose a appropriate method to find the optimum combination. A GA[5] is chosen to meet the requirements in this study.

When the integer coefficients t^i are considered as a gene, a string in the GA can be expressed as $t^1t^2\cdots t^m$. In this case, the boundary ∂_r operator plays the role of decoding the genotype into the phenotype in the GA. The fitness function F for the optimization is defined as

$$F = -\sum_{i=1}^{N}\sqrt{u_i^2 + v_i^2 + w_i^2}, \quad (5)$$

where u_i, v_i and w_i $(i=1, \cdots, N)$ indicate the displacements in the x, y and z directions respectively at the ith loading point. In the case of two-dimensional frames, w_i is equal to zero. The fitness function is defined such that its value increases with increasing frame stiffness. The strings are selected by the strategy of elitist preservation on the basis of the fitness function value ranking calculated by finite element analysis of the frame.

Two methods, A and B, given below, are proposed to satisfy the weight constraint. In method A, the diameter of the members d is constrained by the prescribed total weight of the frame W, the weight per unit volume of the material used γ and the total length of the members L, according to the equation $d = 2\sqrt{W/\pi\gamma L}$. In method B, the diameter of the members d is held constant at 30mm. The upper limit U of the total length of the members L is constrained so that the frame weight is less than the prescribed value W. When the total length L of a certain string is longer than the upper limit U, the fitness function value F is changed to a value $-C-(L-U)$, where $-C$ is a constant much smaller than any likely value of F calculated by Eq. (5). This method is used only for two-dimensional frames. The total length L is discrete, because the nodal coordinates are fixed. This implies that it is impossible to obtain a frame weight exactly equal to the prescribed value W.

4. Numerical examples

It is impossible to prove that a solution obtained by GA is an optimum solution. In order to compensate for this deficiency, a best-first search[6] is added to the GA optimization, that is, the optimization method changes from the GA to a best-first search when the fitness function value of an elitist string is constant for twenty generations. All figures in this section indicate the frame, which has the largest fitness function value in five or six GA test runs. These frames cannot be guaranteed to be optimum in the sense that the sum of their displacements is proved to be a minimum. In the numerical examples, Young's modulus and the specific gravity of the frame material are taken to be 202 GPa and 7.83, respectively.

4.1 Topological optimization of two-dimensional frame

The parameters used in the GA are as follows: population is 434, probability of mutation is 1.0% for each locus, two point crossover being adopted. The number of 2 simplexes used is 144 in the ground structure. By taking advantage of the symmetry of the frame with respect to the horizontal line from the loading point to the rigid wall, half of the 2 simplexes, i.e. 72, is sufficient for the length of a string (the number of loci).

Figure 4 shows the optimum frame obtained from the ground structure shown in Fig. 1 using constraint method A. The prescribed total weight W is 39.2 kg. The fitness function value is -0.104×10^{-2}mm, and the diameter of each member d is 22.7mm.

Figures 5, 6, and 7 show the frames obtained using constraint method B (d=30mm), with the upper limit of length U as 20%, 40% and 60% of the total length of the ground structure of 31.0m, respectively. These length limits are equivalent to weight limits of 34.3 kg, 68.6 kg and 102.9 kg. The total lengths L of the members shown in Figs. 5, 6 and 7 are less than each upper limit U: 19.6%, 39.7% and 59.9% of the length of the ground structure. The values of the fitness function for the frames in Figs. 5, 6 and 7 are -0.290×10^{-2}, -0.739×10^{-3} and -0.477×10^{-3}mm, respectively.

As shown in the figures, the 1-simplexes in the central area near the rigid wall and in the corners on the right side and around the loading point tend to be eliminated from the square design domain of the ground structure.

4.2 Topological optimization of three-dimensional frame

In this section a three-dimensional frame is optimized using constraint method A. The prescribed total weight W is 78.3 kg. The parameters used in the GA and the result obtained are listed in Table 1.

Figure 8 shows the optimum frame under torsional load applied at the upper face of the cube. The

Table 1 Parameters used in GA and sum of displacements (three-dimensional frame)

figure	load (×4)	string	feature value	crossover	length of string	mutation	3-simplex	sum of displacements
Fig. 8	138.6 N	860	-2,-1,1,2	two-point	96	0.5 %	384	0.154×10^{-1} mm
Fig. 13	138.6 N	860	-1,1	two-point	96	0.5 %	384	0.116×10^{-2} mm
Fig. 14	169.7 N	280	-3,-2,-1,1,2,3	simple	24	1.0 %	48	0.334×10^{-2} mm

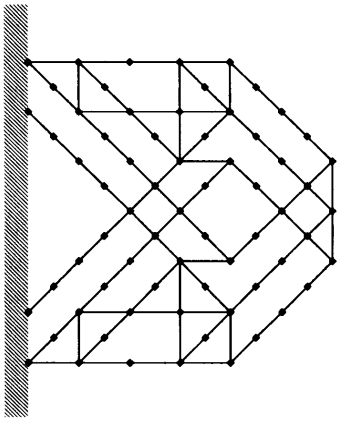

Fig. 4 Two-dimensional frame obtained by constraint method A

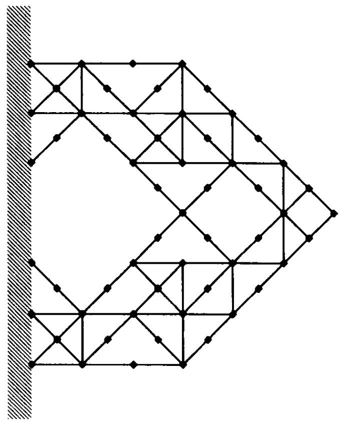

Fig. 6 Two-dimensional frame obtained by constraint method B(40%)

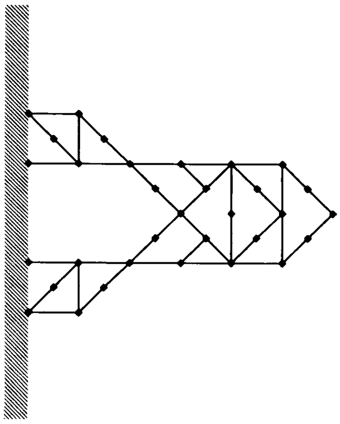

Fig. 5 Two-dimensional frame obtained by constraint method B(20%)

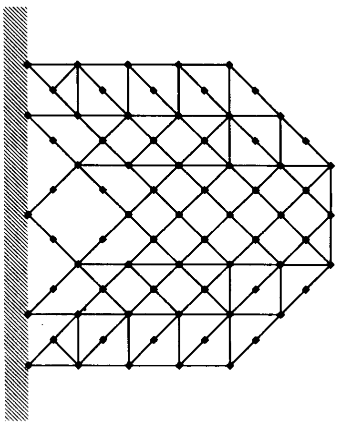

Fig. 7 Two-dimensional frame obtained by constraint method B(60%)

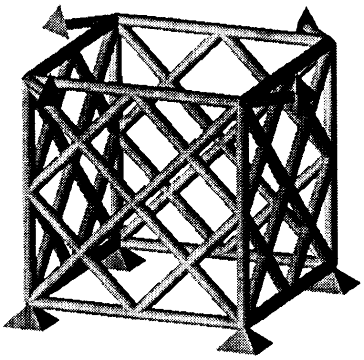

Fig. 8 Frame subjected to torsion on upper face of cube

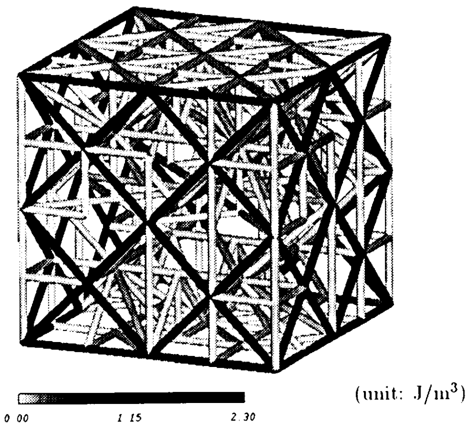

Fig. 9 Distribution of total strain energy density (first generation)

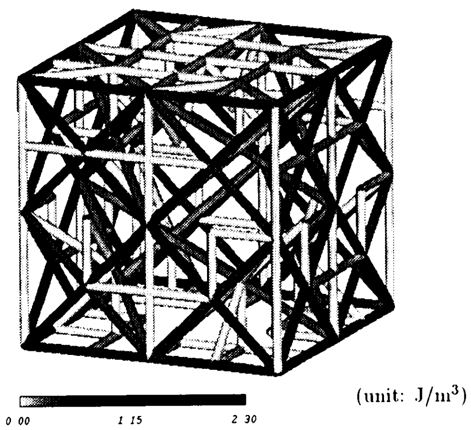

Fig. 10 Distribution of total strain energy density (eighth generation)

arrows in all figures in this section indicate load. The members in the inner space of the cube are eliminated from the ground structure, while some members remain in the four vertical sides of the cube. Owing to the symmetry of the cubic ground structure domain and the loading, integer coefficients t^i are allotted to a quarter of the 3-simplexes. Figure 9 depicts the elitist frame in the population of the first generation, and the total strain energy density in each member is

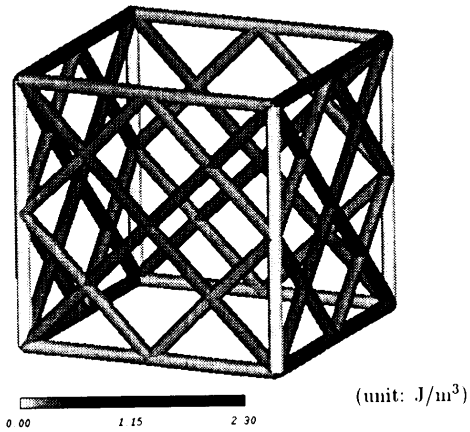

Fig. 11 Distribution of total strain energy density (151st generation)

indicated by the degree of black shading. The diameter of each member of the first generation is calculated as 10.5 mm. It is seen that the total strain energy density in the side surface members of the cube is much larger than that in the inner members. This agrees well with the knowledge inferred from structural mechanics that the inner members do not contribute to the torsional rigidity of the frame.

Figure 10 shows the elitist frame in the eighth generation. The diameter of each member changes to 12.4 mm. The number of members left inside the cube decreases with successive generations, and the difference between the maximum and minimum total strain energy densities is smaller than that of the generation shown in Fig. 9. Figure 11 shows the elitist frame in the 151st generation, that is, the optimum frame shown in Fig. 8. The diameter of each member is 19.2 mm, nearly twice that of the members in Fig. 9. It is shown in Fig. 11 that the total strain energy density is uniformly distributed over all members except the four vertical members at the corner edges of the cube.

Figure 12 shows how the fitness function F (the upper row), the total strain energy (the middle row) and the ratio of the bending strain energy to the total strain energy (the lower row) change as the number of generations increases. The value of the fitness function F increases with successive generations, while the total strain energy decreases. The ratio of the bending strain energy decreases to a small value. It is concluded based on this figure that the optimum frame subjected to torsion carries external load not by bending action but by axial force transmission.

Figure 13 shows the optimum frame obtained when four diagonal forces are applied horizontally so as to enlarge the upper square of the cube. In this case, all members except those on the cube edges are allocated in the upper plane. Figure 14 shows the

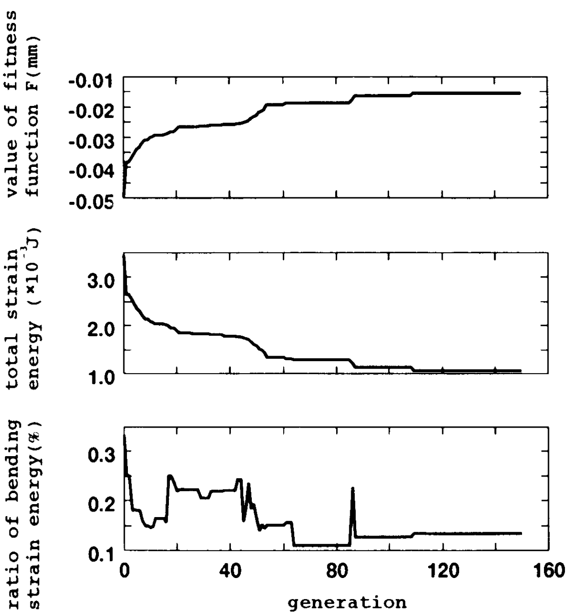

Fig. 12 Evolution of fitness function (top), total strain energy (middle) and ratio of bending strain energy to total strain energy (bottom)

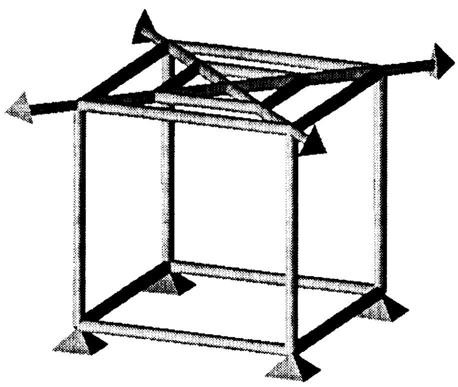

Fig. 13 Frame subjected to diagonal forces on upper face of cube

optimum frame for another loading case, in which upward and downward components are added to the load in Fig. 13. In this frame there are members in all sides of the cube except for the base, since this frame is subject to both diagonal loading in the upper plane and vertical loading. It is necessary to allot integer coefficients t^i to half of the 3-simplexes because the symmetry in this loading case differs from that in Figs. 8 and 13. Consequently, the number of 3-simplexes used is 48, which is fewer than the 384 used in the cases shown in Figs. 8 and 13.

5. Conclusions

A method of topological optimization is proposed on the basis of representing the topology of a frame structure by a combination of integers coded by the boundary cycle and locating the optimum string using

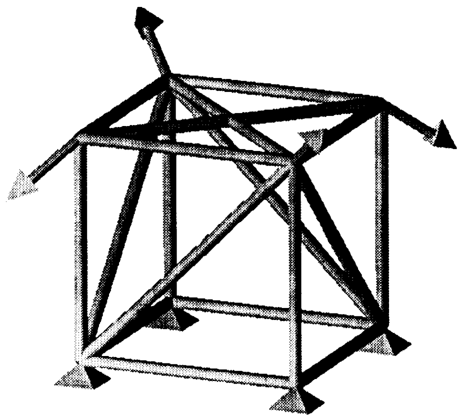

Fig. 14 Frame subjected to upward and downward forces on upper face of cube

a GA. The validity of this method is confirmed by numerical examples concerning optimization of two-dimensional and three-dimensional frames under various loading conditions.

In the optimization process the 1-boundary cycle satisfies the topological constraint that all strings in the GA must represent a one-dimensional complex without a tip. This prevents breeding of useless members which do not contribute to force transmission but increase the frame weight without making any mechanical contribution, and enables efficient location of the best string. In the case of two-dimensional frames, it is found that in the optimum frame there are few members in the central area near the rigid wall and in the corner of the design domain near the loading point, and that most of the members are around the upper and lower sides of the design domain. The optimum three-dimensional frame carries the load not by bending of the members but by axial force transmission.

The integer coefficients of the simplexes that generate the 1-boundary cycle include information relating to both magnitude and orientation, although only the magnitude information, nonzero or zero, is used to determine the existence or nonexistence of the member in this study. It will be possible to represent not only the topology but also the geometry and size by integer coefficients in future when a method of including information such as internal force and member cross section in the integer coefficients has been developed.

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