Two-Dimensional Collisions of Vehicles*
(Case of Consideration of Vehicle Movements during Impact)

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This paper deals with two-dimensional collisions of vehicles based on impulse-momentum relationships. Conventionally, the impact is assumed to occur in an instant, and movements of the vehicles during impact are ignored. In general, however, vehicles are deformed and move in relation to each other throughout the duration of impact. If these deformation and movement are neglected, significant errors occur under certain impact conditions. In this paper, it is assumed that vehicle deformations are characterized by a linear plastic body, and the impact plane and the impact point are determined from the shape of the crushed region at each time step. Collision problems considering vehicle movements during impact can be analyzed using this method. Several results of collision analysis are shown for comparison between this method and the method ignoring vehicle movements. Finally, the vehicle velocities immediately after impact and the vehicle trajectories are discussed.

Key Words: Dynamics of Machinery, Modeling, Simulation, Two-Dimensional Collisions, Vehicle Movements during Impact, Traffic Accident

1. Introduction

Automobile traffic accidents are serious problems in our society. Therefore, in order to deal properly with traffic accidents it is necessary to reconstruct them and to clarify their causes. The field in which engineering knowledge is applied to problems of judicial importace is called 'forensic engineering'. The purpose of this paper is to establish an analytical method for the collisions of vehicles in view of the forensic engineering.

Until now, in the analyses of vehicle collision, vehicles have been assumed to be two-dimensional rigid bodies, and in many cases the impulse-momentum relationships have been applied to them. In the classical collision theories, it has been assumed that the impulse acting between two vehicles is transferred instantaneously through one point. Thus the vehicle movements during impact have been ignored[(1)-(5)]. Actually, however, since vehicles change their positions and shapes during impact, the state of forces acting on the centers of gravity of vehicles also changes. Therefore, vehicle movements during impact cannot be ignored under certain impact conditions[(6)].

In this paper, by dividing impact duration into infinitesimal time steps and applying the impulse-momentum relationships to each time step, we can analyze vehicle impacts with consideration of vehicle movements during impact. In considering the deformations of vehicles due to impact, we assume the vehicles to be linear plastic bodies. By doing so, we can determine the impact center and the direction of impact force from the shapes of crushed regions at each time step.

Finally we compare actual test results[(7)] with results obtained using the present method and the classical method which ignores vehicle movements during impact. We discuss the influences of vehicle movements during impact on vehicle velocities immediately after impact and vehicle trajectories. We calculate the vehicle trajectories after impact using the four-wheel vehicle model[(8),(9)].

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2. Two-Dimensional Collisions of Vehicles

2.1 Impulse-momentum relationships

The following assumptions concerning the vehicle collisions are made:

(1) Forces acting on tires from the road are negligibly small compared with the forces acting between the bodies.

(2) Masses, polar moment of inertia about the center of gravity and sizes of vehicles are known. Also vehicle velocities immediately before impact and the friction coefficient between the bodies are known.

Now we apply the impulse-momentum relationships to two-dimensional vehicle collisions. In the classical collision theories[(1)-(5)], since the duration of impact is very short, it was assumed that the impact occurred in an instant. Therefore, the movements during impact were ignored and the impulse-momentum relationships were applied supposing that the impulse is transferred from the beginning to the end of impact through one point fixed in space. Consequently, when the impact point moves considerably during impact, errors in the magnitude of rotational moments about the centers of gravity of vehicles arise.

In this paper, we divide impact duration into infinitesimal time steps and apply the impulse-momentum relationships to each time step. Here, we update vehicle positions at every time step by assuming that the impact occurs instantaneously in each time step. Thus we can analyze the vehicle collisions with consideration of the change of the state of force transmission due to vehicle movements and deformations.

Here, we consider a collision between two vehicles as shown in Fig. 1(a), where the right front corner of vehicle 1 collides against the right side of vehicle 2 with impact angle γ. We use two coordinate systems as shown in Fig. 1(b). One is a global coordinate system $x-y$ which has a fixed point in space as its origin. The other is a local one $t-n$ which has the impact point (x_c, y_c) as its origin. The axis t is directed along the impact plane and the axis n is perpendicular to t. The angle between t and x axes is denoted by α_c. Vehicle direction angles (taken from the y axis and clockwise direction positive) and the coordinates at the centers of gravity are denoted by, $\alpha_1, (x_1, y_1)$, and $\alpha_2, (x_2, y_2)$ for vehicles 1 and 2, respectively.

We can describe the relationships between x and y components of impulse increments, ΔP_x, ΔP_y, and t and n components, ΔP_t, ΔP_n, as follows:

$$\begin{aligned}\Delta P_x &= \Delta(P_t\cdot\cos\alpha_c)-\Delta(P_n\cdot\sin\alpha_c)\\ \Delta P_y &= \Delta(P_t\cdot\sin\alpha_c)+\Delta(P_n\cdot\cos\alpha_c)\end{aligned}\tag{1}$$

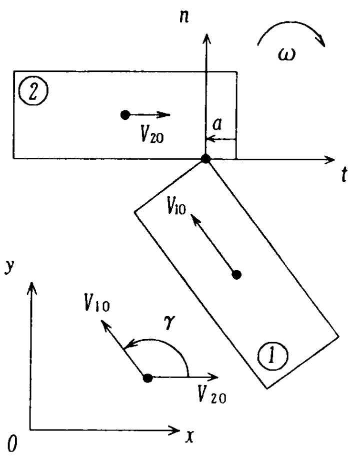
For vehicles 1 and 2 at any time, we obtain the impulse-momentum relationships in the x-, y-directions and rotation about the center of gravity in incremental form as follows:

$$\begin{aligned}m_1\cdot\Delta u_1 &= -\Delta P_x\\ m_1\cdot\Delta v_1 &= -\Delta P_y\\ j_1\cdot\Delta\omega_1 &= \Delta(P_x\cdot b_1)-\Delta(P_y\cdot a_1)\\ m_2\cdot\Delta u_2 &= \Delta P_x\\ m_2\cdot\Delta v_2 &= \Delta P_y\\ j_2\cdot\Delta\omega_2 &= -\Delta(P_x\cdot b_2)+\Delta(P_y\cdot a_2)\end{aligned}\tag{2}$$

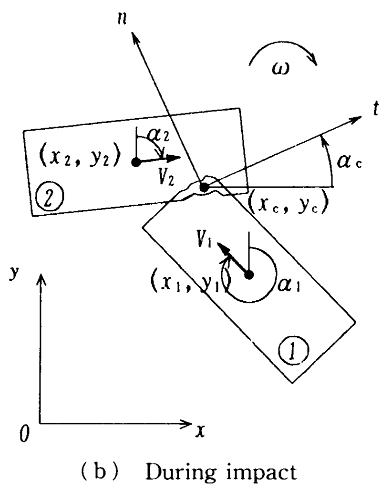
where m is vehicle mass, j is polar moment of inertia about the center of gravity, u, v are x, y components of vehicle velocities, respectively, ω is angular velocity, a, b are the lengths given by Eq.(3), and subscripts 1 and 2 refer to vehicles 1 and 2, respectively.

It has been assumed in the classical collision theories[(1)-(5)] that a and b remain constant during impact. In this paper, to take the vehicle movements during impact into consideration, we update a and b at each time step by using the following equations.

$$\begin{aligned}a_1 &= x_1-x_c,\quad b_1=y_1-y_c\\ a_2 &= x_2-x_c,\quad b_2=y_2-y_c\end{aligned}\tag{3}$$



(a) Beginning of impact



(b) During impact

Fig. 1 Impact of two vehicles

Using the above relationships, we can obtain the velocity increments at the centers of gravity in the infinitesimal time step from the impulse increments, ΔP_t and ΔP_n, coordinates of the centers of gravity, (x_1, y_1), (x_2, y_2), the impact point (x_c, y_c) and the angle of the impact plane, α_c.

2.2 Transmission of forces at the impact point

We separate the force acting at the impact point into two components. These are frictional force acting along the impact plane and compressive force acting on the impact plane perpendicularly.

(a) Frictional force We apply Coulomb's law to the friction at the impact point. The sliding direction is determined from the sign of the sliding velocity S. The sliding velocity S at an arbitrary time can be expressed in terms of the velocities of the centers of gravity by using the following equation.

$$S=\{(u_1-b_1\cdot\omega_1)-(u_2-b_2\cdot\omega_2)\}\cos\alpha_c+\{(v_1+a_1\cdot\omega_1)-(v_2+a_2\cdot\omega_2)\}\sin\alpha_c \qquad (4)$$

The relationship between the impulse increments is given as

$$\Delta P_t=\frac{S}{|S|}\mu\Delta P_n, \qquad (5)$$

where μ indicates the friction coefficient between two bodies.

(b) Compressive force To consider the vehicle deformation due to impact, we assume the vehicle strength to be isotropic and approximate the vehicle body as a linear plastic body. The impact force is transmitted through one point in each time step as mentioned in section 2.1. Therefore, we consider a linear plastic spring along the n axis. Thus the compressive force of the spring arises as the impact force F_n along the n axis.

We regard the area of overlap of the two vehicles formed at each time step (see Fig. 1(b)) as the sum of the deformations of the two vehicles. We call this area the 'total crushed region' hereafter. Assuming the magnitude of F_n is proportional to the area of the total crushed region, the increment of F_n in each time step, ΔF_{ni}, is given by

$$\Delta F_{ni}=K\cdot\Delta s_i,\ K=\frac{k_1\cdot k_2}{k_1+k_2}, \qquad (6)$$

where Δs_i is the area of the total crushed region in each time step, k is the stiffness of linear plastic spring which relates the area of the plastic crushed region of each vehicle to the compressive force F_n as shown in Fig. 2, and subscripts 1 and 2 refer to vehicles 1 and 2, respectively.

Accordingly, we can calculate the components of impulse increments ΔP_l from the following equations.

$$\Delta P_l=F_l\cdot\Delta t,\ F_l=\sum_{i=1}^{m}\Delta F_{li} \quad (l=x, y) \qquad (7)$$

where Δt is the infinitesimal time step and m is the number of time steps to the arbitrary time.

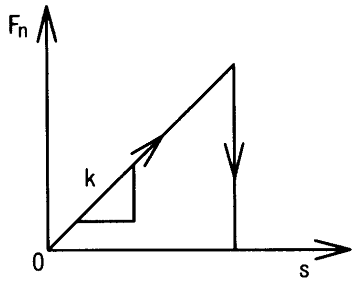


Fig. 2 Linear plastic spring

2.3 Vehicle deformation during impact

As we mentioned in the previous section, the magnitude of compressive force occurring at the impact point is proportional to the area of the crushed region. From Eq.(6) and the relation, $\Delta s_i=\Delta s_{1i}+\Delta s_{2i}$, we obtain the relationships between areas of crushed regions at each time step and the vehicle stiffnesses as

$$\Delta F_{ni}=k_1\cdot\Delta s_{1i}=k_2\cdot\Delta s_{2i}. \qquad (8)$$

We can rewrite the above expressions into the next form.

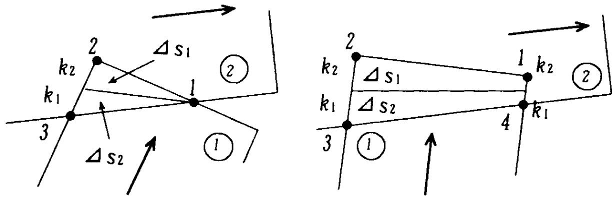
$$\Delta s_{1i} : \Delta s_{2i}=k_2 : k_1 \qquad (9)$$

The total crushed area Δs_i is partitioned into the crushed areas for each vehicle, Δs_{1i} and Δs_{2i}, according to Eq.(9).

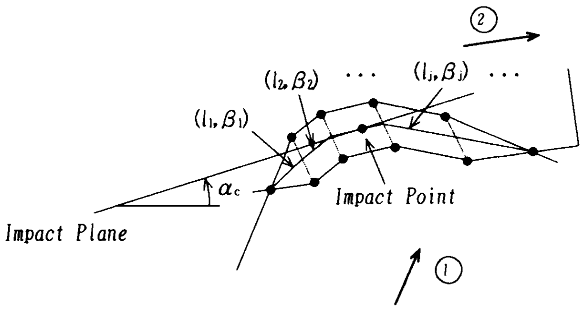
The method of partitioning of total crushed area is described in the following paragraphs.

We consider the first time step in impact calculation. In this case, the shape of the total crushed region becomes triangular or quadrilateral depending on the velocities of vehicles and the impact angle γ. Figure 3(a) shows the case of a triangular deformation. We consider a point which divides side $\overline{23}$ interiorly into $k_2 : k_1$ and draw a line which links this point to point 1. We regard this line as the interface between two vehicles. Next, Fig. 3(b) shows the case of a quadrilateral deformation. We consider two points which divide sides $\overline{23}$ and $\overline{14}$ interiorly into $k_2 : k_1$, respectively and draw a line which links these two points. We regard this line as the interface between two vehicles.

In and after the second time step, the shapes of the total crushed region become polygonal in general. In this case, as shown in Fig. 3(c), first, we consider the lines which partition the total crushed region into rectangles or squares and we make points which divide these lines interiorly into $k_2 : k_1$, respectively. Then we construct a folded line which links these points. We regard this line as the interface between two vehicles. We cannot strictly satisfy the relations in Eq.(9) in these manners. However, since the smaller the time step we take, the smaller the total crushed region at each time step becomes, if we select



(a) Triangle (b) Quadrilateral



(c) Polygon

Fig. 3 Deformation of vehicle during impact

a sufficiently small time step, we can satisfy the relations approximately.

The crushed regions of the two vehicles increase in area throughout the duration of impact. However, when the areas of the crushed regions cease to increase, the impact ends, and the two vehicles separate. Hence the condition of impact end is expressed as

$$\Delta s_i = 0. \tag{10}$$

2.4 Impact point and impact plane

Since the impact force is transferred through one point in each time step as mentioned in section 2.1, we need to determine the applying point (the impact point) and the direction of impact force (the angle of the impact plane).

In this paper, we consider the impact point as the intersection between the interface of vehicles and the line linking the centers of gravity of crushed regions of each vehicle. Also we take the angle of the impact plane as the average gradient of the vehicle interface.

Because the vehicle interface becomes a folded line due to the procedure described in section 2.3, we calculate the angle of the impact plane from the average gradient of each line segment weighted by each length as follows.

$$\alpha_c = \frac{\Sigma(l_j \beta_j)}{\Sigma l_j} \quad (j = 1, 2, 3, \cdots) \tag{11}$$

where l_j, β_j indicate the lengths and the gradients of the line segments, respectively.

2.5 Vehicle movements during impact

Vehicle velocities at an arbitrary time are calculated by adding the velocity increments until that time to the initial velocities.

$$u = u_0 + \sum_{i=1}^{m} \Delta u_i$$
$$v = v_0 + \sum_{i=1}^{m} \Delta v_i \tag{12}$$
$$\omega = \omega_0 + \sum_{i=1}^{m} \Delta \omega_i$$

where subscript 0 refers to the initial velocity and m indicates the number of time steps to the arbitrary time.

The coordinates of the center of gravity in the global coordinate system can be calculated by adding the coordinates of the center of gravity (x_0, y_0) and the direction angle α_0 at the beginning of impact to the vehicle movements until that time as shown in the following equations.

$$x = x_0 + \sum_{i=1}^{m} \Delta x_i = x_0 + \sum_{i=1}^{m} (u_i \cdot \Delta t)$$
$$y = y_0 + \sum_{i=1}^{m} \Delta y_i = y_0 + \sum_{i=1}^{m} (v_i \cdot \Delta t) \tag{13}$$
$$\alpha = \alpha_0 + \sum_{i=1}^{m} \Delta \alpha_i = \alpha_0 + \sum_{i=1}^{m} (\omega_i \cdot \Delta t)$$

2.6 Analytical procedure

The analytical procedure of this method is as follows:

(1) Set the values with respect to vehicles, initial velocities and all other values required in the analysis.
(2) Calculate ΔP_n from the area of the total crushed region in the time step (Eqs. (6) and (7)).
(3) Checking the sign of the sliding velocity S, calculate (Eq. (5)).
(4) Determine the impact point and the impact plane from the shape of the crushed region (Eq. (11)).
(5) Calculate ΔP_t the velocity changes of the centers of gravity in the time step (Eq. (2)).
(6) Update the vehicle velocities and positions of the centers of gravity (Eqs. (12) and (13)).
(7) Determine the end of impact by checking the condition of impact end (Eq. (10)).
(8) Repeat steps (2)-(7) to impact end.

3. Numerical Examples

We compare the results obtained using the present method considering the vehicle movements during impact with results obtained using by the classical theory[1], and with the experimental results[7]. Here, we applied the four-wheel vehicle model[8],[9] to computation of vehicle trajectories after impact. The trajectories after impact are depicted at intervals of 0.5 sec until 2 sec.

Also, we carry out the impact analyses varying the impact angle. We show the differences between

Table 1

	vehicle 1	vehicle 2
masses (kg)	$m_1=1949$	$m_2=1189$
polar moments of inertia (kgm^2)	$j_1=4704$	$j_2=2263$
lengths(m)	5.3	3.9
widths(m)	2.0	1.5
wheelbases(m)	3.0	2.0
front treads(m)	1.8	1.3
rear treads(m)	1.8	1.3
tire-road friction coefficient	0.87	
steering angle (deg)	0	
wheels	unlocked	
impact velocities of vehicles (m/s)	$V_{10}=V_{20}=9.6$	
impact angle (deg)	$\gamma=120$	
impact location(m)	$a=1$	
friction coefficient between the bodies	$\mu=0.5$	
stiffnesses of plastic springs (kN/m^2)	$k_1=k_2=350$	
time step (ms)	$\Delta t=5$	

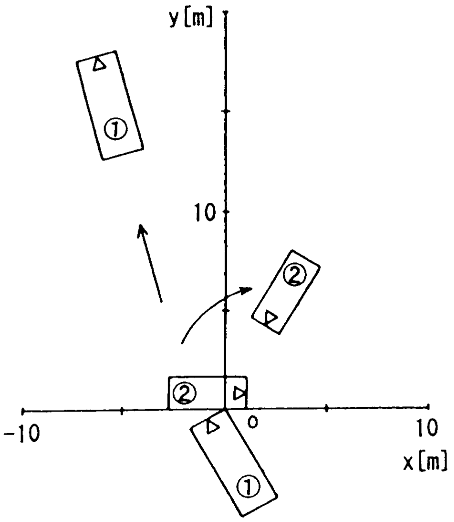


Fig. 4 Experimental results

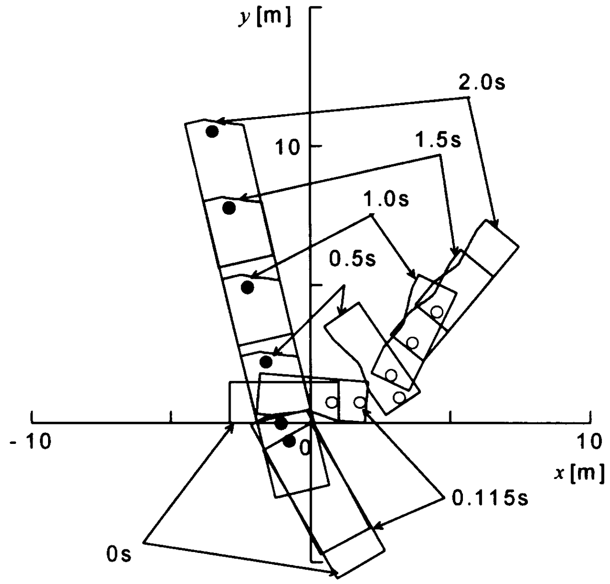


Fig. 5 Trajectories until 2 sec obtained using the present method

the vehicle velocities immediately after impact calculated from the present method and those calculated from the rigid body collision theory(1).

3. 1 Comparison with the experimental results

The values and conditions used in the analysis are shown in Table 1.

We set the stiffness of plastic spring making reference to Ref.(10). The stiffnesses of both vehicles are assumed to be equal. The coefficient of restitution in the rigid body collision analysis is set to zero.

Results from the experiment, the present method and the rigid body collision theory are shown in Figs. 4, 5 and 6, respectively. Vehicle behaviors during impact obtained by the present method are shown in Fig. 7.

We find from Figs. 5 and 6 that the rotations of vehicle 2 are considerably different between the results of the present method and the rigid body collision theory. In this example, we set the impact point on the right side of the center of gravity of vehicle 2 at the beginning of impact. Since it is assumed in the rigid body collision theory that the impact force is transferred at this point throughout the duration of impact, a rotational moment in the counterclockwise direction arises about the center of gravity of vehicle 2 (Fig. 6). However, it is clear from the experimental result (Fig. 4) that vehicle 2 is rotated in the clockwise direction after impact. This difference is caused by the movement of the impact point during impact. That is, although the impact

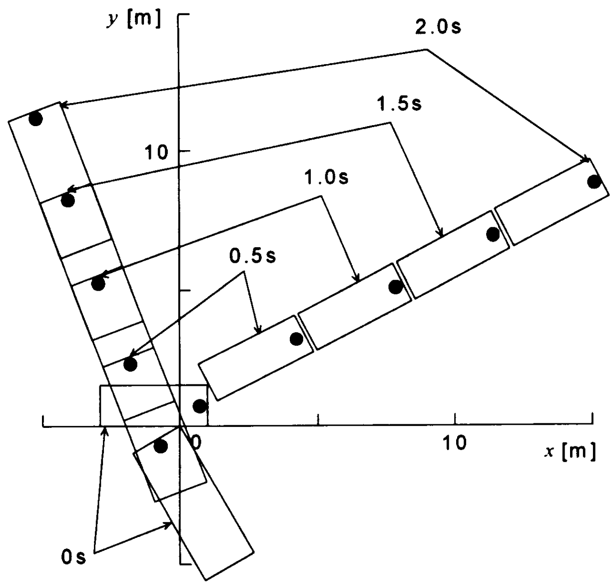


Fig. 6 Trajectories until 2 sec obtained using the rigid body collision theory

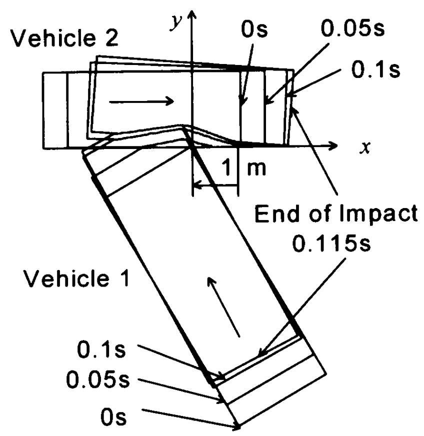
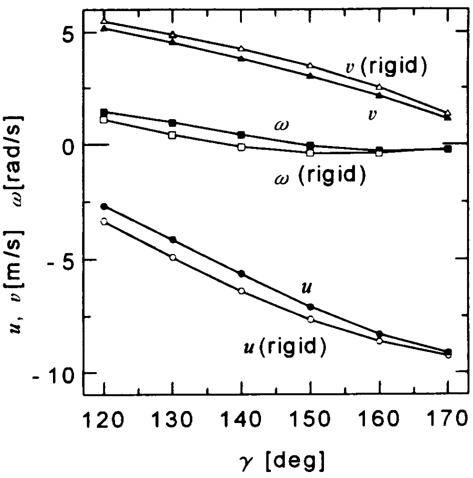
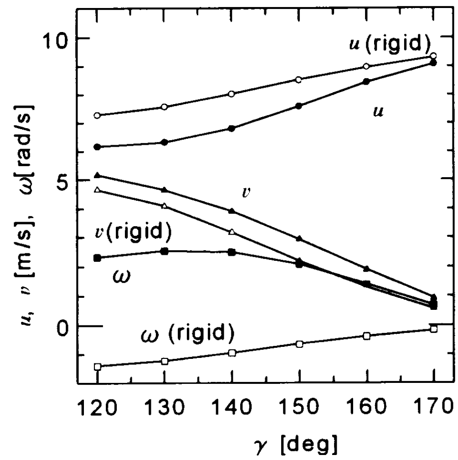


Fig. 7 Vehicle behavior during impact (γ=120 deg, a=1 m)



(a) Vehicle 1



(b) Vehicle 2

Fig. 8 Vehicle velocities immediately after impact (a=1 m, γ=120 - 170 deg)

point is located on the right side of the center of gravity of vehicle 2 at the beginning of the impact, it moves to the left side of the center of gravity of vehicle 2 during impact. Therefore, the clockwise rotation is given to the center of gravity of vehicle 2 finally. This phenomenon is simulated by the present method as shown in Figs. 5 and 7.

The present method can simulate the change of impact force due to movement of the impact point. Therefore, it is expected that the accuracy of the solution can be improved by setting more appropriate stiffnesses.

3.2 Case of varying the impact angle

It was found from the comparison in the previous section that when the vehicle movements during impact are large, the results of the rigid body collision theory contains errors in the rotation of vehicle 2 in particular. To examine this tendency in detail, we carry out the impact analysis using both the present method and the rigid body collision theory, varying the impact angle from 120 to 170 deg and we compare the vehicle velocities immediately after impact. The values and the conditions used in the analyses are the same as those described in section 3.1. The results of both analyses are shown in Fig. 8.

It is seen in Fig. 8(b) that the differences between the results of the two theories are apparent with respect to the angular velocity of vehicle 2. The differences increase when the impact angle becomes small. On the other hand, when the impact angle becomes large, the differences become small though the vehicle movements are large. This is because the impulse transferred and the rotational moments applied to the center of gravity of vehicle 2 are small.

4. Conclusions

The authors analyzed vehicle impacts with consideration of vehicle movements during impact, dividing impact duration into infinitesimal time steps and applying the impulse-momentum relationships to each time step. In considering the deformations of vehicles due to impact, we approximated the vehicles to be perfectly plastic. We determined the impact center and the direction of impact force from the shapes of crushed regions at each time step.

Finally we compared the results obtained using the present method with those of the rigid body collision theory and with experimental results. The following were found from these comparisons:

(1) the impact analysis based on the rigid body collision theory results in errors in angular velocity of the vehicle 2 when the impact point moves considerably during impact (particularly, when the direction of rotational moments about the center of gravity of the vehicle 2 changes), and

(2) the differences in vehicle positions and velocities immediately after impact considerably influence the vehicle trajectories after impact.

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